2008/6

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Equilibria in markets with non-convexities and a solution to the missing money phenomenon in energy markets

Gabriella Muratore

CORE
Voie du Roman Pays 34
B-1348 Louvain-la-Neuve, Belgium.
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
E-mail: corestat-library@uclouvain.be
http://www.uclouvain.be/en-44508.html

CORE DISCUSSION PAPER
2008/6

Equilibria in markets with non-convexities and a solution to the missing money phenomenon in energy markets

Gabriella MURATORE[1]

February 2008

Abstract

In this paper we address the issue of finding efficient partial equilibria in markets with non-convexities. This is a problem that has intrigued generation of economists. Beside its theoretical importance this issue is fundamental in energy markets which do not give the right price signals and incentives to maintain existing and invest in new generating capacity. By considering a competitive environment in which consumers maximize utility independently of other agents actions while suppliers are profit maximizers given other market agents actions, we are able to find efficient prices in markets with non-convexities. Based on this result we propose a design for an energy-only market able to give investors the correct price signals.

Keywords: energy markets, equilibrium prices, non convex economies.

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[1] Technische Universiteit Eindhoven, The Netherlands. E-mail: g.muratore@tue.nl

This paper was partially done while I was visiting CORE, Louvain-la-Neuve, whose all staff I wish to thank for their friendly attitude. My most deepest thanks are, though, for Prof. Jacques Drèze without whose moral support this paper would have never been written. I would also like to thank him for the many suggestions, useful comments and nice discussions we had. Special thanks should be given to Pierre Dehez and Yves Smeers for suggesting relevant literature. This research was supported by a grant from University of Catania.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.

1 Introduction

Finding clearing prices and quantities in markets with non-convexities is enjoying a renewed popularity thanks to the deregulation of the electricity sector in many countries which has brought forth classical market mechanisms for the efficient allocation of demand/supply. In particular, energy markets have been designed under the assumptions of perfect competition and operated accordingly. Regardless of the specific implementation and operative procedures, all energy markets are characterized by a *day-ahead* (and/or a *hour-ahead*) market mechanism. There, suppliers and demanders bid their respective schedules for the next day (and/or hour) and a market maker set clearing prices and quantities. The market mechanism can be as simple as intersecting the energy supply curve with the demand curve or as complex as finding the dual prices (nodal prices) of an optimization program which keeps in account, beside bids, all operative and network constraints. After many years of operation, though, there is an increasing evidence and corresponding policymakers concern that wholesale energy markets (regardless of the chosen market design) are not providing enough economic incentives to stimulate investments in new generating capacity nor in existing one. This problem is known in the literature and in specialized journals and circles as the "resource adequacy problem", the "supply security problem" or as the "missing money problem" since the main issue really is that energy prices are not high enough to cover investments costs. Many authors have been investigating the phenomenon, providing both possible explanations of its causes and ingenious market designs to cope with it. For example, Hogan, [21], and partially Cramton and Stoft, [11], claim that the missing money phenomenon is caused by the price caps imposed by regulators to mitigate market power while Joskow, [24], notes that price caps are usually not even binding in most U.S. wholesale energy markets that exhibit "missing money". Citing Joskow, [24], "*Even during scarcity hours market prices are below the price caps. Accordingly, it is unlikely that the price cap are the only source of the missing money problem"*. Joskow, [24], and Joskow and Tirole, [23], support the view that prices are kept low during scarcity hours by some hidden system operator behavior and protocols, e.g., system voltage reduction, out of market contracts (OOM), reliability must run (RMR). On the other hand all these authors recognize that lack of demand response and the impossibility to route electricity flow to single customers are the primary reason for system operator protocols. In the end all authors agree that generators need to receive more from the market, or through energy-only high enough payments, [21], or through no capped energy spot market combined with forward capacity market, [11] and [24]. From a more theoretical standpoint most of the work for finding efficient prices in energy markets has concentrated on two-part tariffs or in the pricing of different "commodities" like operative reserve and capacity. Non linear pricing as been well researched, see [36] for a comprehensive analysis, but it has always been considered appropriate only in not perfectly competitive market environments. More recently, O'Neill et al., [28], have applied non linear pricing in a competitive market environment (with a special focus to energy market) by explicitly pricing non convexities as it were different types of commodities. The "missing money phenomenon" in the energy industry, though, is not a new problem in the economic literature. It was first analyzed and brightly solved by the French marginalist school, in particular by Boiteux, [7], executive at EDF,[1] the French nationalized electricity utility. See Dréze, [15], for a very good review of the contributions of French economists to theory and public policy. Because of the industry cost structure, characterized by large fixed costs and relatively smaller variable costs, marginal pricing is not enough to guarantee recovery of total costs, even in a regulated monopoly. The above mentioned author proved that, when capacity is tight, i.e., at peak load, marginal price has to increase by the capacity cost per unit of the marginal capacity in order to recover total costs. Peak load pricing is quite easy to implement in a centrally managed, regulated industry but it has not found yet the proper adaptation in a competitive market framework. This is because it has been widely believed that no efficient clearing price will exists in a competitive market in the presence of non convexities either in the cost functions or in the production sets. After the seminal work of Arrow and Debrew, [5], who have proved the existence of competitive equilibrium prices for a general economic system under convexity assumptions (and, hence, excluding the most common situation of increasing return of scale), the problem has been amply studied by economists who have derived quite interesting results under the most diverse economic settings, principles and mathematical hypothesis. We should remind here the work of Guesnerie,[19], who characterizes Pareto optimality in non convex economies, of Beato, [6], who discusses the existence of equilibria under marginal cost pricing in differentiable economies displaying increasing return of scale, Brown et al., [8], who generalize some of these results, and of Dehez and Dréze, [12], who have proved the existence of general competitive equilibria in the non convex case under the assumption of voluntary trading: *"... producers announce prices for their*

[1] Electricite´ de France

outputs and satisfy the demand which materialize at these price ...". Their approach is based on objecting the neoclassical assumption that firms maximize profit at given price: they, instead, turn over the assumption and introduce the notion of quantity-takers suppliers. This is in line with the "Critique of Principles" of Neoclassical theory, started around 1925 with Sraffa, [33], [34]. He disputed that the long term cost functions can have the usual U-shape under the classical assumption of perfect competition. The "Critique" continued afterwards with the empirical work of Hall and Hitch, [20]. They analyzed businessmen behavior and concluded that businessmen are not maximizing profits as the neoclassical theory dictates but they are more concerned about *full cost* and what they can actually sell on the market. The theory of *full cost pricing* will be developed afterwards by Andrews, [1], [2], [3],[4], Sylos Labini, [35], Edwards, [17], Kalecki, [25], Eichner, [18], Moss, [27] and others. For these authors prices are determined by the production cost under normal utilization of the plants plus a mark-up. Even those that support the idea that profit maximization is the firm goal, do recognize that there are some inconsistency within the neoclassical theory. "*...In particular, long run profit maximization does no imply, in general, equality between short run marginal cost and marginal revenue*, [37]. This issue was discussed at length in the Oxford Economic Papers, between 1954 and 1956, by authors like H. F. Lydall, M. E. Paul, J. Hicks, P. Streeten, F. H. Hahn, H. R. Edwards. Building on all these contributions we too mildly criticize the neoclassical assumptions underling the perfectly competitive market model, in particular, the assumption that agents maximize their own utility without any consideration of other agents actions. A simple example, reported in the next section, shows that this assumption will lead to no market equilibrium even in very simple cases. In fact, Arrow and Debrew, [5], in order to prove the existence of an equilibrium for the convex case, do need to assume that some agents (in their case, consumers) actions depend on other agents actions. In this paper we use the definition of an abstract economy as in [5] but, instead of assuming that consumers maximize utility given producers choices (which determine after all consumer available budget) while producers maximize profit subject only to their own technological constraints, we turn around the hypothesis and assume that consumers maximize utility independently of other agents, under an exogenously determined budget, while producers maximize profit under their own technological (and financial) constraints given other agents actions. In particular, the aggregate production of other firms combined with the level of market demand characterize the demand faced by each firm. In our view this assumption more closely represents real economic systems where the number of consumers is much larger than the number of firms which, in the end, need to take into account both the level of market demand and other agents cost structure and production possibilities. This approach can be seen as analogous to searching for supply constrained equilibria or underemployment equilibria, a well researched topic in economic literature, see Roberts, [30], Dehez and Dréze, [14], Citanna et al., [10]. Also, in the partial equilibrium setting (which is of the most importance to us because of our application to energy markets) it is standard to assume consumers budget exogenously given. Under this framework, by describing suppliers production sets via a cost function (not necessarily convex) and a compact set (non necessarily convex) of possible production outputs, we are able to prove not only the existence of Pareto efficient equilibrium prices but also to provide a method for computing them. We will apply our findings to the energy industry and sketch an efficient market design for an energy-only market able to restore the "missing money". This is, by necessity, not to make the paper unduly long and unreadable, just a sketch for a possible energy market design which does not take into account all the operative constraints of a real setting. Also, we should mention here the fact that our efficiency result depends on the market maker collecting all relevant and truthful information from the participants, in particular each producer's cost structure. Hence, the market maker need to have perfect information to achieve efficiency, a result in line with Hurwicz findings, [22], and this delineates the difficulty of the underlying problem and the limits (to efficiency) implied by current, implemented schemes. We will discuss at length all these issues in our forthcoming paper. We will leave to future research the issue of finding equilibrium prices for even more general economic models. The paper is organized as follows: in the next section we briefly remind the neoclassical assumptions underlying competitive markets and give a simple example that shows how, even under the best of circumstances, a market equilibrium may fail to exists if all agents act independently of each other. Then we define a competitive economy in which the set of strategies available to some agents, particularly producers, is affected by other agents choices. At this point we are able to prove the existence of efficient, partial equilibria in markets with non convexities, both in the short and in the long run and to give algorithms to actually compute them. By a simple argument these results can also be applied to find equilibrium prices in multiple markets with independent demands where suppliers are producing all or some of the commodities. In the last section we show, through examples taken from the literature, how to apply these theoretical results to real markets, in particular energy markets, and propose an efficient energy-only market design able to restore the missing money in a competitive way.

2 The perfectly competitive market model

Perfect competition is an idealized economic model whose main feature is that of *no market power* i.e., where no single economic agent has the power to influence the price through its behavior and where collusion is not possible. This situation is summed up by saying that *all agents are price takers*. This fundamental requirement is coupled with the following other assumptions that, all together, define the *competitive market model*, as introduced by Cournot and developed by neoclassical economists:

1. No market power: agents are price takers

2. Homogeneity: all producers (supply agents) sell a homogeneous product or service

3. Equal access: all firms have access to all available production technologies and resources

4. No barriers: all agents can enter and exit the market as they wish

5. Perfect information: all agents have complete and symmetric information

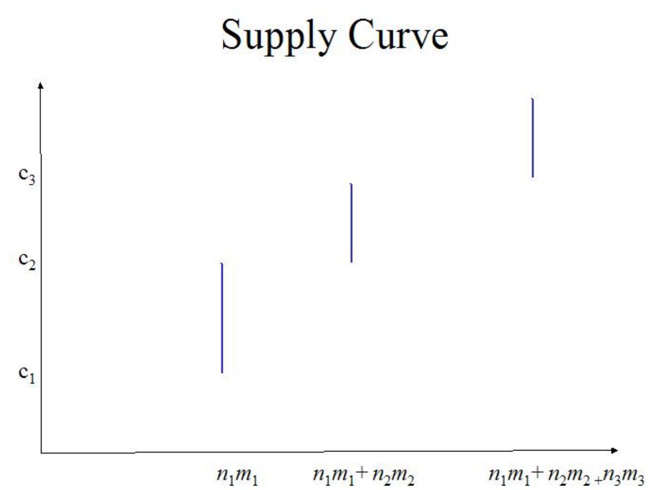
The behavior of each agent is that of maximizing its own utility, that, for firms, coincide with profit. In the classical setting, agents are assumed to behave *independently* of others so that their decision to produce and/or consume depends solely by the *given* market price and by their own production and budget constraints. Given the price, total supply equals the sum of each producer supply and total demand equals the sum of each consumer demand. The market or equilibrium price is, by definition, the price at which total supply equals total demand. The first thing we should notice is that, in the short run, when the technology of each firm (and hence its capacity) is fixed by definition, these assumptions are not enough to assure the existence of an equilibrium even in the convex case. Suppose, in fact, that we have a finite number of technologies, let say k and, within each, a finite and large number of identical firms, n_i for $i = \{1, \ldots, k\}$. Suppose that each firm within technology i can produce any amount between 0 and its maximum capacity m_i (convex production set) and that each firm has a linear (hence convex) cost function $F_i + c_i q_i$ defined by a fixed cost F_i and a constant marginal cost c_i. Given the market price p^*, the firm will solve the following optimization problem:

$$\begin{aligned} Max\ \Pi(q_i) &= p^*q_i - c_iq_i - F_i = (p^* - c_i)q_i - F_i \\ s.t. & \\ 0 \leq q_i \leq m_i & \end{aligned}$$

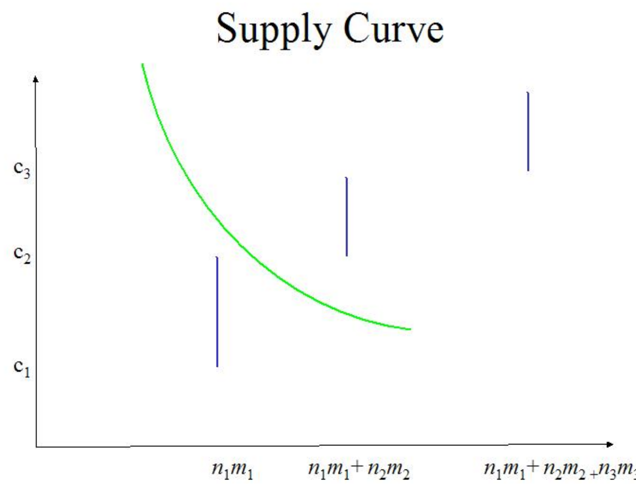
whose solution is as follow:

$$\begin{aligned} q_i &= 0 \quad \text{if } p^* \leq c_i \\ q_i &= m_i \quad \text{if } p^* > c_i \end{aligned}$$

When $p^* = c_i$ all points in the production interval will give the same result, namely, a loss equal to the fixed cost and, hence, the firm may well not produce at all. Without loss of generality, let's assume that $c_1 \leq c_2, \leq \ldots, \leq c_k$. Given a price p^*, total supply will be equal to $S = \sum_{j=1}^{h} n_j m_j$ where $h = max\{i/c_i < p^*\}$. The market supply function will be a discontinuous function with a finite number of vertical pieces as shown in figure.



Hence, the intersection between the demand curve and the supply curve may fail to occur, as in the next picture.



The next Theorem gives a sufficient condition for the existence and uniqueness of a (partial) market equilibrium.

Theorem 2.1 *Let $S(p)$ be the market supply as a function of the price and $D(p)$ the market demand as a function of the price. Let $\hat{p}$ be the minimum price at which the firms will supply the market, i.e., $S(\hat{p}-\alpha) = 0$ for all $\alpha > 0$. If $S(\hat{p}) \leq D(\hat{p})$, $S(p)$ is a continuous, non decreasing function of the price, $D(p)$ is a continuous, strictly decreasing function of the price then it exists a unique market equilibrium. Similarly, if $S(\hat{p}) \leq D(\hat{p})$, $S(p)$ is a continuous, strictly increasing function of the price, $D(p)$ is a continuous, non increasing function of the price then it exists a unique market equilibrium.*

The above theorem is the underlying reason why so much economic literature has focused on proving that, indeed, demand and supply curve have this shape in most markets. If, actually, this is the case for the demand function, we can not say the same for the supply function as the previous simple example proves. In order to assure the existence of an equilibrium point in a competitive market without making restrictive assumptions about cost functions and supply curves we need to drop the unrealistic assumption that in a competitive market all agents maximize utility without any consideration of other agents behavior. At least some of them need to take into consideration others agents behavior. This is actually what Arrow and Debreu needed to assume in their famous work *Existence of an Equilibrium for a Competitive Economy*. In this work, they generalize the notion of a game and define an *abstract economy* with l products and r agents as follows:

Definition 2.2 *An Abstract Economy is the tuple*

$E = (\Omega_1, \Omega_2, \ldots, \Omega_r, f_1, f_2, \ldots, f_r, A_1(\overline{a}_1), \ldots, A_r(\overline{a}_r))$ *where* $\Omega_i \subseteq R^l$, $\Omega = \Omega_1 \times \Omega_2 \times \ldots \times \Omega_r$, $\overline{\Omega}_i = (\Omega_1 \times \Omega_2 \times \ldots \times \Omega_{i-1} \times \Omega_{i+1} \times \ldots \times \Omega_r)$, f_i *is a real function defined over* Ω, $f_i : \Omega \rightarrow R$, *while* $A_i(\overline{a}_i)$ *is a multi-valued function defined over* $\overline{\Omega}_i$ *whose values are subsets of* Ω_i, $A_i : \overline{\Omega}_i \rightarrow \wp(\Omega_i)$, *i.e.,* A_i *associates to any* $(r-1)$*-tuple in* $\overline{\Omega}_i$ *a subset in* Ω_i.

This definition can be seen as a generalization of a *game* in which r individuals, upon choosing a strategy, resulting in a strategy profile $(\hat{a}_1, \ldots, \hat{a}_r)$, such that $\hat{a}_i \in A_i(\hat{a}_1, \ldots \hat{a}_{i-1}, \hat{a}_{i+1}, \ldots \hat{a}_r)$ for all i, i.e., $\hat{a}_i$ belongs to the set of available action to player i, will receive the payoff $f_i(\hat{a}_1, \ldots, \hat{a}_r)$. The difference with a standard game is that in this latter the set of available strategies to each player is independent of other players choices. Hence, in an abstract economy, the choice of an agent affects both the payoff and the feasible set of actions of other players. It is, then, only natural to define an equilibrium for an abstract economy E as:

Definition 2.3 *An equilibrium for an abstract economy is a* $r-$*tuple* $(a_1^*, a_2^*, \ldots, a_r^*)$ *such that* $a_i^* \in A_i(\overline{a}_i^*)$ *and* $f_i(\overline{a}_i^*, a_i^*) = max_{a_i \in A_i(\overline{a}_i^*)} f_i(\overline{a}_i^*, a_i)$.

An economic equilibrium is characterized by the fact that each agent is maximizing its own utility given the actions of other agents. These other agents actions may or may not affect the feasible set of actions available to a given agent. In the same fundamental work, [5], Arrow and Debreu define an abstract economy in which l commodities are traded and agents are divided among consumers, producers and a fictitious agent, called the *market participant* or *market maker* who chooses prices and whose behavior reflects the law of supply and demand, i.e., the price of a commodity rises if demand exceeds supply and falls otherwise. The payoff to a consumer is given by its utility function, to a firm by its profit and to the market maker by balancing demand and supply in all markets. Consumers and market maker available actions do depend on other agents actions while producers available actions are assumed to be independent by other agents actions. It is then proved that, under suitable assumptions, among which there is, remarkably, the assumption of non-increasing return of scale, an equilibrium exists. We will follow track, but instead of assuming that consumers set of actions depends on other agents actions we will assume that consumers act independently while the set of actions available to each producer does depend on other agents actions. In particular, the demand faced by each firm does depend on overall market demand and other firms aggregate production. The assumption in this terms seems just obvious.

In the next section we will address the existence of an equilibrium for an abstract economy where a single commodity is traded (partial equilibrium) and where increasing return of scale and in general, non convexities, are present. We will explore both the single and multi period setting (short and long run).

3 Partial Equilibria in Competitive Markets with Non Convexities

Let us consider a competitive market with the following features:

1. Homogeneity: all producers (supply agents) sell a homogeneous product or service.

2. No market power and no collusion : agents are price takers.

3. Equal access: all firms have access to all available production technologies and resources.

4. Perfect information: all agents have complete and symmetric information.

5. No barriers: all agents can enter and exit the market as they wish. In particular, firms will exit the market if they are not making at least the normal profit in a given time horizon and they will enter the market if they can make at least the normal profit in a given time horizon. Firms staying in the market for a given time period can decide not to produce at any point in time: they will always do so if the market price at a given time is such that revenues do not cover their variable costs.

6. Consumers maximize their own utility independently of all other agents subject only to their own budget constraint. Each firm maximizes his own utility, in the face of a firm's demand that depends on market demand as well as on other firms aggregate production, (i.e., producers are maximizers, given other agents actions), subject to their own production (technological) and budget (financial) constraints.

We will distinguish two cases: in the first case there is a single period, within which all firms need to achieve the normal profit or will not enter the market; in the second case the given time horizon T is divided in t periods.

The problem is then that of finding equilibrium prices $(p_1^*, p_2^*, \dots, p_t^*)$ that equates supply and demand in each period and such that each firm will earn the normal profit within the given time horizon T.

3.1 The Single Period

1. Consumers. There are m consumers, each with a utility function $u_i : R^+ \rightarrow R$ and with a given budget B_i. At a given price p^*, each consumer, independently of all others agents, will ask the quantity d_i^* which solves the following optimization problem:

$$\max u_i(d)$$
$$s.t.$$
$$d \in X_i(p^*)$$

   where $X_i(p^*)$ is the feasible set at price p^* for consumer i, $X_i(p^*) = \{d \geq 0 : p^* d \leq B_i\}$.

   Note that, at the given price p^*, the total demand is $D(p^*) = \sum_{i=1}^{m} d_i^*$.

2. Technologies. We assume there are k different technologies that can produce the same homogenous output and whose cost structure is given by cost functions $C_i(q)$ for $q \in K_i \subseteq R^+$ (K_i is the production possibility set of technology i and it is any subset of the non negative real numbers). $C_i(q)$ is by definition the minimum cost at which the amount q can be efficiently produced by technology i. Without loss of generality we can assume that $C_i(q) = C_i^F + C_i^V(q)$ where $C_i^F \geq 0$ is the fixed cost, incurred independently of the amount produced, while C_i^V is the variable costs that, instead, does depend on the amount produced. Within technology i there are, potentially, n_i equal firms, where n_i is a large number, modelling no barrier to entry. Not all these firms need to be active at equilibrium, modelling no barrier to exit. At equilibrium, we will have the optimal number n_i^* of firms within technology i. Actually, we can also use this same framework to model the short run when the number of firms n_i is fixed. In this case the maximum capacity of technology i is $n_i m_i$, its fixed costs is $C_i^F = r_i C_i^F$, where $r_i \leq n_i$ is the number of active firms, and its variable cost is $C^V(Q) = r_i C^V(\frac{Q}{r_i})$. In order to simplify notation, in what follows we will be referring to technologies, keeping in mind, though, that a supplier should be a firm j within technology i. We will indicate by $\tilde{K}_i$ the production possibility set of technology i as a whole. Each

technology will maximize its own profit subject to its own technological and financial constraints and by facing a technology demand which depends by market demand (i.e. consumers preferences) and by the aggregate production of other technologies, i.e., depends by others agents actions. In particular, a technology will produce nothing if it can not achieve at least the normal profit.

3. Market Maker. There is a market maker which set the price and whose goal is to equate supply and demand in such a way to achieve Pareto efficiency given other players actions.

Our market economy can be defined as:

$$E = (R^+, R_1^+, \ldots, R_m^+, R_1^+, \ldots, R_k^+, u_1, \ldots, u_m, \Pi_1, \ldots, \Pi_k,$$

$$X_1(\overline{a}_1) \ldots, X_m(\overline{a}_m), A_1(\overline{a}_1), \ldots, A_k(\overline{a}_k))$$

where R^+ is the set of non negative real numbers. The first component represents the price (and hence the market maker), the following components from 1 to m represent the demand of each consumer, the following components from 1 to k represent the supply of each technology. u_i is the utility function of consumer i and depends solely by its own demand d_i:

$$u_i(p, d_1, \ldots, d_m, Q_1, \ldots, Q_k) = u_i(d_i)$$

Π_j is the profit of technology j and depends only by the price and its own produced quantity (and its own cost structure):

$$\Pi_j(p, d_1, \ldots, d_m, Q_1, \ldots, Q_m) = \Pi(p, Q_j)$$

X_i is the available set of consumption pattern to consumer i and depends solely by the price (and consumer i budget):

$$X_i(p, d_1, \ldots, d_{i-1}, d_{i+1}, \ldots, Q_1, \ldots, Q_k) = \{d \geq 0 : pd \leq B_i\}$$

Finally, A_j represents the available set of production levels for technology j and does depend on all other agents actions (as well by its own technological and financial constraints):

$$A_j(p, d_1, \ldots, d_m, Q_1, \ldots, Q_{j-1}, Q_{j+1}, \ldots, Q_k) =$$

$$\{Q > 0 : Q \leq \sum_{i=1}^{m} d_i - \sum_{l \neq j} Q_l, Q \in \tilde{K}_j, \Pi_j = pQ - C^T(Q) \geq 0\} \cup \{0\}$$

Any (non zero) element in $A_j(.)$ is the portion of *"residual"* demand, given other agents actions, that technology j can satisfy under its own constraints. If the residual demand is zero, i.e., other agents are already fulfilling all market demand, the only available strategy for the agent is not producing. At the given price p^* technology i is solving the following problem:

$$\begin{aligned} \text{Max } \Pi_i(p^*; Q_i) \\ s.t. \\ Q_i \in A_i(\overline{Q}_i) \end{aligned}$$

Definition 3.1 *An equilibrium in our competitive market is a vector* $(p^*, d_1^*, \ldots, d_m^*, Q_1^*, \ldots, Q_k^*)$ *such that*

1. $p^* \geq 0$
2. $\sum_{i=1}^{m} d_i^* = \sum_{i=1}^{k} Q_i^*$
3. $d_i^* \in X_i(\overline{d_i^*}) = X_i(p^*)$ *and* $u_i(d^*) = max_{d \in X_i(p^*)} u(d)$ *for all* $i = \{1, \ldots m\}$
4. $Q_j^* \in A_j(\overline{Q_j^*})$ *and* $\Pi(Q_j^*) = max_{Q \in A_j(\overline{Q_j^*})} \Pi(Q)$ *for all* $j = \{1, \ldots k\}$

A competitive partial equilibrium, hence, gives a price that equates supply and demand and such that consumers are maximizing utility under budget constraint and producers are maximizing profits under production and financial constraints given other agents actions. The definition does not exclude multiple equilibria. We are looking for Pareto (or otherwise efficient) equilibria. First, we will consider the case in which all suppliers have non increasing average total cost functions and then we generalize our findings to include general cost functions. Next, we will describe an optimization problem whose solution, under some assumption, will give a Pareto efficient equilibrium. Consider the following optimization problem P:

$$\max \int_0^Q D^{-1}(x) d(x) - \sum_{i=1}^{k} \sum_{j=1}^{n_i} C_{ij}^T(q_{ij}) z_{ij}$$

$$s.t.$$

$$
\begin{aligned}
Q &= \sum_{i=1}^{k}\sum_{j=1}^{n_i} q_{ij} && && (1)\\
p &= D^{-1}(Q) && && (2)\\
pq_{ij} - C^T_{ij}(q_{ij})z_{ij} &\geq 0 && \forall\ i = \{1,\dots,k\},\ \forall\ j = \{1,\dots,n_i\} && (3)\\
q_{ij} &\in K_i && \forall\ i = \{1,\dots,k\},\ \forall\ j = \{1,\dots,n_i\} && (4)\\
q_{ij} &\leq Mz_{ij} && \forall\ i = \{1,\dots,k\},\ \forall\ j = \{1,\dots,n_i\} && (5)\\
z_{ij} &\in \{0,1\} && \forall\ i = \{1,\dots,k\},\ \forall\ j = \{1,\dots,n_i\} && (6)\\
p &\geq 0 && && (7)
\end{aligned}
$$

where

- $D^{-1}(Q)$ is the market demand inverse function. By definition, the market demand is the locus of points such that consumers are maximizing their own utility (under budget constraints) at given prices.

- $C^T_{ij}(q_{ij}) = C^T_i(q_{ij})$ is the total cost of supplying q_{ij} units of product by using technology i incurred by firm j. It includes fixed costs, if any.

- K_i is the feasible production set for technology i. Since it is always feasible not to produce, $0 \in K_i$ for all i.

- z_{ij} are decision variable that can take only $\{0,1\}$ values. If z_{ij} is set to zero, the corresponding firm j adopting technology i will not produce and it may or may not incur its fixed cost: this will depend on the type of problem we are modelling. If it is an investment decision then a non-active firm will not incur its fixed cost; if, though, fixed costs are already sunk the firm will incur its fixed costs. In either case, the cost of non active firms is not reflected, rightly, in the market allocation cost. If z_{ij} is set to 1 it will produce an amount such that at the market price of our single period model its revenues will be greater or equal to its total costs. This profitability requirement is expressed through constraint (3).

- Constraint (4) just express technological feasibility of production. Taken together, constraints (3) and (4) represents each firm financial and technological constraints.

- Constraint (5) is needed to force production q_{ij} to be zero at firm j adopting technology i if the corresponding decision variable $z_{ij} = 0$. M is a large number. It is enough to take this number equal to the maximum capacity among the technologies.

- Constraints (1) and (2) equates total demand and total supply at a given price p.
- The objective function is just the sum of each player utility function since $\int_0^Q D^{-1}(x)d(x) =$ *Consumer surplus* + *Total Market Revenue* = *Consumers surplus* $+ pQ$ and the term $\sum_{i=1}^{k}\sum_{j=1}^{n_i} C_{ij}^T(q_{ij})z_{ij}$ is the total cost of supplying Q amount of product to the market. Hence, the objective function can be read as *Consumers surplus*+ *Total Market Profit* i.e., the usual *Welfare Function*.

Here n_i is a large number for each i, representing no barrier to entry and exit the market. The optimal solution to problem P will, hence, give the optimal number of firms within each technology. Note that if an optimal solution to the foregoing optimization exists then there exists another optimal solution where $q_{ij} = q_{il}$ for $j, l \in \{1, \ldots, n_i\}$ and for any technology i. Hence, w.l.g. we will assume throughout this paper that the optimal solution of problem P is of this latter form.

Theorem 3.2 *If the utility function of each consumer, $u_i(q)$, is such that $u_i(q) < u_i(q+k)$ for any positive $k > 0$, the market demand strictly decrease for any price increase and each producer j has non increasing average total cost then, if a solution of the problem P exists, it is a Pareto efficient solution for the competitive market described by assumption* $1-6$.

Proof. Let us suppose that an optimal solution $S^* = (p^*, Q^*, q_{11}^*, q_{12}^*, \ldots q_{kn_k}^*)$ to the foregoing optimization problem exists. It is equivalent to $\tilde{S}^* = (p^*, d_1^*, \ldots, d_m^*, q_{11}^*, q_{12}^*, \ldots q_{kn_k}^*)$ where $u(d_i^*) = \max\{u_i(d) : d \in X_i(p^*)\}$ since by definition of market demand $\sum_{i=1}^m d_i^* = D(p^*) = Q^*$. Let us prove that the solution is indeed Pareto optimal for the game at hand. By contradiction, suppose that there exists another output of the game, $\hat{S} = (\hat{p}, \hat{d}_1, \ldots, \hat{d}_m, \hat{q}_{11}, \ldots \hat{q}_{kn_k})$, where $\sum_{i=1}^m \hat{d}_i = \sum_{(ij)} \hat{q}_{ij} = \hat{Q}$ and $\hat{p} = D^{-1}(\hat{Q})$, such that all agents are not worse off than in S^* and at least one agents is strictly better off. Let us call by $A^* = \{(ij)/z_{ij}^* = 1\}$ i.e., A^* is the set of all firms that are supplying the market according to solution S^*. If $\hat{p} > p^*$ then by our assumption about market demand we have that $\hat{Q} < Q^*$ and hence some consumer needs to cut off his/her consumption and he/she will be definitely worse off than in solution S^* because of the non saturation hypothesis. Hence, $\hat{p} \leq p^*$. Suppose that $\hat{p} < p^*$. It follows that $\hat{Q} > Q^*$ and consumer surplus strictly increases. If Market profit strictly decreases than in S^*, i.e., $\sum_{(ij)} \Pi(\hat{p}; \hat{q}_{ij}) < \sum_{(ij)} \Pi(p^*; q_{ij}^*)$ then some agent in A^* is definitely worse off in $\hat{S}$ than in S^*. If, instead, $\sum_{(ij)} \Pi(\hat{p}; \hat{q}_{ij}) \geq \sum_{(ij)} \Pi(p^*; q_{ij}^*)$

then some constraint of problem P must be violated by $\hat{S}$, namely constraint 3 or 4, otherwise S^* would not be the optimal solution to problem P. If constraint 4 is violated then the allocation in $\hat{S}$ is not physically feasible and should be discarded. Suppose, hence, that solution $\hat{S}$ is violating only constraint 3 i.e., the firms financial constraint. It must be, though, that the financial constraint is violated only for firms outside the set A^*. In fact, $\Pi(\hat{p};\hat{q}_{ij}) \geq \Pi(p^*;q^*_{ij}) \geq 0$ for all $(ij) \in A^*$ since, otherwise, some of these firms is definitely better off with output S^*. This in turn implies that $\hat{q}_{ij} > q^*_{ij}$ for all $(ij) \in A^*$ since $\hat{p} < p^*$ and average total cost is non increasing in quantity. To see why this must be the case, suppose that $\hat{q}_{ij} < q^*_{ij}$. From $\Pi(\hat{p};\hat{q}_{ij}) \geq \Pi(p^*;q^*_{ij}) \geq 0$ it follows that

$$\frac{\hat{p}\hat{q}_{ij} - C_{AT}(\hat{q}_{ij})\hat{q}_{ij}}{\hat{q}_{ij}} > \frac{\hat{p}\hat{q}_{ij} - C_{AT}(\hat{q}_{ij})\hat{q}_{ij}}{q^*_{ij}} \geq \frac{p^*q^*_{ij} - C_{AT}(q^*_{ij})q^*_{ij}}{q^*_{ij}}$$

and, hence, $\hat{p} - C_{AT}(\hat{q}_{ij}) > p^* - C_{AT}(q^*_{ij})$ which implies $C_{AT}(\hat{q}_{ij}) < C_{AT}(q^*_{ij})$ against our hypothesis on average total costs. Finally, it is easy to see that can not be $\hat{q}_{ij} = q^*_{ij}$. Hence $\sum_{(ij)\in A^*} \hat{q}_{ij} > \sum_{(ij)\in A^*} q^*_{ij} = Q^*$. The point $\overline{S} = (\overline{p}, \overline{Q}, \overline{q}_{11}, \ldots, \overline{q}_{kn_k})$ where $\overline{Q} = \sum_{(ij)\in A^*} \hat{q}_{ij}$, $\overline{p} = D^{-1}(\overline{Q})$, $\overline{q}_{ij} = \hat{q}_{ij}$ for all $(ij) \in A^*$ and $\overline{q}_{ij} = 0$ otherwise, is feasible to problem P since $\Pi(\overline{p};\overline{q}_{ij}) = \overline{p}\hat{q}_{ij} - C^T_{ij}(\hat{q}_j) \geq \hat{p}\hat{q}_{ij} - C^T_{ij}(\hat{q}_j) = \Pi(\hat{p};\hat{q}_{ij}) \geq \Pi(p^*;q^*_{ij}) \geq 0$ for all $(ij) \in A^*$ since $\overline{Q} \leq \hat{Q}$ and hence $\overline{p} \geq \hat{p}$. Moreover, $\overline{S}$ has an objective value strictly better than S^* since $\overline{Q} > Q^*$ implying $\overline{p} < p^*$ implying further a strictly higher consumer surplus than in S^* and $\sum_{ij} \Pi(\overline{p};\overline{q}_{ij}) = \sum_{(ij)\in A^*} \Pi(\overline{p};\hat{q}_{ij}) \geq \sum_{(ij)\in A^*} \Pi(p^*;q^*_{ij}) = \sum_{ij} \Pi(p^*;q^*_{ij})$. This contradicts the fact that S^* was the optimal solution to problem P. Finally, if $\hat{p} = p^*$ then $\hat{Q} = Q^*$ and from the above it follows that $\hat{q}_{ij} = q^*_{ij}$ for all $(ij) \in A^*$ and hence $S^* = \hat{S}$. ■

Lemma 3.3 *Let us assume that each supplier total cost function is differentiable in the interior of the supplier production set and that there exist left and right derivatives at the boundary points.*
Let $S^* = (p^*, d^*_1, \ldots, d^*_m, q^*_{11}, q^*_{12}, \ldots q^*_{tn_k})$ *be an optimal solution to problem* P *and let* A^* *be the set of active suppliers at this optimal solution. If* $p^* \geq max\{C'_i(q) : q \in A_i(\overline{q^*_{ij}})\}$ *for all* $(ij) \in A^*$ *then* S^* *is an equilibrium for the competitive market described by assumption* $1-6$.

Proof. The result follows by noting that, under the hypothesis, is $p^*(q_{ij} + h) - C^T(q_{ij} + h) \geq p^*q_{ij} - C^T(q_{ij})$ for any $q_{ij} \in A_i(\overline{q}^*_{ij})$ and any $(ij) \in A^*$ and, hence, each supplier optimal action is to fulfill the whole residual demand, i.e., $q^*_{ij} = \sum_{i=1}^{m} d^*_i - \sum_{t=1}^{k}\sum_{l=1}^{n_t} q^*_{tl}$ with $(tl) \neq (ij)$, is such that $\Pi(q^*_{ij}) = \max\{\Pi(q) : q \in A_{ij}(\overline{q}^*_{ij})\}$. ■

We also have the following necessary condition for equilibria:

Lemma 3.4 *Let us assume that suppliers cost function are differentiable in the interior of their production set. If $S^* = (p^*, d^*_1, \ldots, d^*_m, q^*_{11}, q^*_{12}, \ldots q^*_{kn_k})$ is an equilibrium for the competitive market described by assumption $1-6$ then $p^* \geq C^{'}_i(q^*_{ij})$ for all active suppliers such that q^*_{ij} is interior to the production set.*

Proof. The result immediately follows since, otherwise, q^*_{ij} would not be a maximization point given other agents actions. ■

Theorem 3.5 *Under the assumption of Theorem (3.2), if an optimal solution to problem P exists then it is a Pareto efficient equilibrium for the competitive market described by assumption $1-6$.*

Proof. By the Theorem (3.2) if a solution to problem P exists then it is Pareto efficient. Let us prove that it is indeed a market equilibrium as defined by (3.1). Conditions $1, 2, 3$ are satisfied through constraints (7),(1),(2). Given other players actions, each firm is maximizing profit over the set of its available strategies since, by hypothesis, total average cost function is not increasing and, hence, the corresponding marginal cost function is below it and by Lemma (3.3) the result follows. ■

The problem P will usually have a solution. In the next theorem we will give sufficient conditions, usually met in real life situation, for a solution to exists. These conditions require basically that the market price can rise high enough if supply is scarce, that total capacity is finite and that it exists a single firm that could act as a monopolist or, in more simple terms, that can supply at some market price $\hat{p}$ the corresponding quantity $D(\hat{p})$ without incurring losses. As one can see, if this last condition is not satisfied there can not be any market whatsoever.

Theorem 3.6 *Suppose that the demand function is continuous as well as each supplier total cost function. Suppose further that each technology set, K_i, is compact and that $\lim_{q\to 1^+} D^{-1}(q) = +\infty$. If it exists a single firm that can supply an amount $\hat{q} \in K_i$ such that $\hat{p} = D^{-1}(\hat{q}) \geq C^{AT}(\hat{q})$ then a solution to problem P exists.*

Proof. Under the hypothesis, problem P feasible set is not empty. Let us consider the price $\hat{p} = max_i\{max_{q\in K_i} C^{AT}_i(q)\}$ where $C^{AT}_i(q)$ is the average total cost function of technology i. At price $\hat{p}$ all firms will supply any

amount so that the equilibrium price is surely less or equal than $\hat{p}$. Hence, w.l.g., we can bound the price by $\hat{p}$ to obtain a bounded feasible set. For each possible combination of active ($z_{ij} = 1$) or non active ($z_{ij} = 0$) suppliers we get a continuous objective function and an empty or a non empty, compact feasible region for which the maximum exists. Since the possible combinations of active and non active suppliers are finite the result follows.
$\blacksquare$

Let us consider now the following function:

$$f(Q) = \min \{\sum_{i=1}^{k}\sum_{j=1}^{n_i} C_{ij}^T(q_{ij})z_{ij} \text{ s.t. } q_{ij} \in S(Q) \text{ for all } (ij)\}$$

where $S(Q)$ is the set described by the following system:

$$\begin{array}{rcll}
Q & = & \displaystyle\sum_{i=1}^{k}\sum_{j=1}^{n_i} q_{ij} & \\
p & = & D^{-1}(Q) & \\
pq_{ij} - C_{ij}^T(q_{ij})z_{ij} & \geq & 0 & \forall\ i = \{1,\ldots,t\},\ \forall\ j = \{1,\ldots,n_i\} \\
q_{ij} & \in & K_i & \forall\ i = \{1,\ldots,t\},\ \forall\ j = \{1,\ldots,n_i\} \\
q_{ij} & \leq & Mz_{ij} & \forall\ i = \{1,\ldots,t\},\ \forall\ j = \{1,\ldots,n_i\} \\
z_{ij} & \in & \{0,1\} & \forall\ i = \{1,\ldots,t\},\ \forall\ j = \{1,\ldots,n_i\} \\
p & \geq & 0 &
\end{array}$$

It is obvious, then, that a solution to problem P can be found by solving

$$\max \quad F(Q) - f(Q)$$

where $F(Q) = \int_0^Q D^{-1}(x)d(x)$ and Q is such that $f(Q)$ is defined i.e., its associated feasible set is non empty. If the function $G(Q) = F(Q) - f(Q)$ is strictly quasi-concave a simple algorithmic scheme would be to find the maximum $\hat{Q}$ such that $f(\hat{Q})$ is defined i.e., the maximum $\hat{Q}$ for which a feasible allocation of demand $\hat{Q}$ among suppliers at price $p(\hat{Q})$ exists, and then computing $G(Q)$ for decreasing value of $Q \leq \hat{Q}$ till a local optimum is reached. Dichotomous or Fibonacci search or other smarter search methods could be used in order to converge faster to the optimum. In most practical cases, though, when the demand function has the usual, downward sloping, convex shape and suppliers average total cost functions are non increasing, the optimal Q^* will be near the maximal feasible $\hat{Q}$.

The algorithm can be seen as a modified version of the Tatonnement process for the Walrasian auctioneer.

1. The auctioneer will cry prices in ascending order.
   (a) At the given price $\hat{p}$, a market demand $D(\hat{p})$ will form by summing the demand of each single consumer.
   (b) At the given price $\hat{p}$, each producer h will compute its supply set $S_h(\hat{p}) = \{q \in K_h : \hat{p}q - C^T(q) \geq 0\}$ and a market supply set $S(\hat{p})$ will form by making the union of all supply sets S_h. Suppliers will bid their cost function along with their supply sets.
2. The auctioneer will compute a min cost allocation of market demand $D(\hat{p})$ over the supply set $S(\hat{p})$ and will then compute total welfare. If total welfare increases with respect to the previous price the process is repeated otherwise the process ends.

Under suitable assumption, as we have seen before, the process will converge to a Pareto efficient equilibrium.
If suppliers average total cost is any function and fixed costs are not already sunk (as in investment decisions) then we still have a Pareto efficient solution since suppliers either recover total costs or will not enter the market. If, though, fixed costs are already sunk we may loose Pareto efficiency. In this case we could use the concept of *Second Best* or the Ramsey-Boiteux prices. The theory of second best refers to situations in which there is some departure from economic efficiency (e.g., taxes, budget constrains, monopoly power, incomplete markets, etc.) and aims at finding optimal policies that can cope with this source of inefficiency. The Ramsey-Boiteux prices are the second best approach to maximize welfare subject to a budget constraint (usually a zero or no loss constraint) and are usually used in public monopolies to avoid financing deficits through taxes. See [7], [16], [32]. Hence, we may claim that, when suppliers average total cost is any function, a solution to problem P is second best or constrained Pareto efficient. It may not be, though, an equilibrium. In order to find efficient equilibria we need to consider additional constraints. Let us consider the problem $P^{'}$ which is the same as P with the addition of the following constraint:

$$(p - C_{ij}^{'}(q_{ij}))(q_{ij} - l_{ij})z_{ij} \geq 0$$

for all (ij)[2] where l_{ij} is the minimum load[3] for technology i. The constraint imposes the necessary condition that, for active suppliers ($z_{ij} = 1$), equilibrium price has to be greater or equal to marginal cost at interior equilibrium quantity. The condition is also sufficient if supplier marginal cost is quasi-monotone. We then get the following results:

Corollary 3.7 *Let us assume that each supplier total cost function is differentiable in the interior of the supplier production set and that there exist left and right derivatives at the boundary points. Let us assume further that each supplier cost function is such that either total average cost is not increasing or marginal cost is quasi-monotone. If a solution to problem P' exists then it is an equilibrium for the competitive market described by assumption $1-6$ and it is second best Pareto efficient with respect to any other possible equilibrium.*

Also, analogous to Theorem (3.6), we get:

Corollary 3.8 *Suppose that the demand function is continuous, that $\lim_{q \to 1^+} D^{-1}(q) = +\infty$, that each supplier total cost function is differentiable in the interior of the supplier production set and that there exist left and right derivatives at the boundary points. Under the additional hypothesis that each technology set, K_i, is compact and that there exists a single firm that can supply an amount $\hat{q} \in K_i$ such that $\hat{p} = D^{-1}(\hat{q}) \geq C^{AT}(\hat{q})$ and $(\hat{p} - C_i'(\hat{q})(\hat{q} - l_i) \geq 0$ then problem P' has a solution.*

The Tatonnement process for the Walrasian auctioneer is the same as before except that now each supplier will bid a supplier set comprised of all the quantities that are profitable to produce at the cried price and such that marginal cost at each of these quantity is non greater than the cried price. Let $\overline{P'}$ be the optimization problem obtained from P' by removing the profitability requirement and by adding the following constraint: $pl_{ij} \geq C^V(l_{ij})$ whenever $q_{ij} = l_{ij}$ and $l_{ij} > 0$ is the minimum load of technology i.

$$\max \int_0^Q D^{-1}(x)d(x) - \sum_{i=1}^{k} \sum_{j=1}^{n_i} C_{ij}^T(q_{ij}) z_{ij}$$

$$s.t.$$

[2]Actually, it is enough to consider only those suppliers whose average total cost is not non- increasing

[3]l_{ij} can also be zero if technology i can produce any $\epsilon > 0$

$$
\begin{aligned}
Q &= \sum_{i=1}^{k}\sum_{j=1}^{n_i} q_{ij} && && (8)\\
p &= D^{-1}(Q) && && (9)\\
(pl_{ij} - C^V(l_{ij})z_{ij} &\geq (q_{ij} - l_{ij})(Neg)\ z_{ij} && \forall\, i,j,\ \ l_{ij} > 0 && (10)\\
(p - C'_{ij}(q_{ij}))(q_{ij} - l_{ij})z_{ij} &\geq 0 && \forall\, i,j && (11)\\
q_{ij} &\in K_i && \forall\, i,j && (12)\\
q_{ij} &\leq Mz_{ij} && \forall\, i,j && (13)\\
z_{ij} &\in \{0,1\} && \forall\, i,j && (14)
\end{aligned}
$$

Neg is a large, negative number.

Corollary 3.9 *Under the assumption of Corollary (3.7), if a solution to problem $\overline{P'}$ exists then it is an equilibrium and it is Pareto efficient with respect to any other possible equilibrium.*

Corollary 3.10 *Under the assumption of Corollary (3.8) a solution to problem $\overline{P'}$ exists.*

3.2 The Multi Period

In this section we analyze the case in which firms need to achieve profit in a given time horizon of T periods or will not produce in any period. We can see this as the problem faced by a firm wishing to enter a given market or as the steady state for already active firms. We will be assuming perfect and complete information as well as perfect foresight.

1. Consumers. In each period t there are m_t consumers with utility function $u_i^t : R^+ \to R$ and an exogenous budget[4] B_i^t which both do not depend on other periods consumption. Hence, market demand in each period is independent.

2. Technologies There is a finite number of technologies that may or may not be present in some period. In the latter case the production possibility set of technology i at time t is empty. This allows to model obsolescence of technology and entrance of new technologies. Also, since

[4]Here we assume that budget in each period does not depend on other period consumption since we are in a partial equilibrium setting and hence budget in period t can also depend on other markets prices.

the production possibility set is indexed by time we can model down-time and other short term change in production possibility as well as investments in capacity and other long term resources. Hence, we can model both short and long run equilibria. The cost structure of each technologies is $C_i^{tot}(q_1, q_2, \ldots, q_T) = C^V(q_1, q_2, \ldots, q_T) + \sum_{t=1}^T C_{ti}^F = C^V(q_1, q_2, \ldots, q_T) + C_i^F$. In each period, within technology i, there are n_i equal firms where n_i is a large number modelling no barriers. Each firm is maximizing profits within the time horizon, given other agents action, with the aim to cover total cost within the time horizon. If this can not be achieved, firm will not produce in any period. Note that without this assumption on profitability we just have a competitive model in which all firms are maximizers given other agents actions. On the other end, for T large enough, a no-loss requirement seems just a necessary economic condition.

3. The Market Maker There is a market maker whose goal is to equate supply and demand in each period and to reach Pareto efficiency given other players actions.

The market economy in this case is represented by:

1. A vector of prices $(p_1, \ldots p_T)$;

2. A vector of consumer demands $(d_{11}, \ldots, d_{Tm_T})$ such that $u_i(d_{1i}, \ldots, d_{Ti}) = \sum_{t=1}^T u_{ti}(d_{ti})$ for any consumer i and $d_{ti} \in X_{ti} = \{d \geq 0 \ : p_t d \leq B_{ti}\}$ for any consumer i and any period t;

3. A vector of production quantity $(q_{1ij}, \ldots, q_{Tij})$ for any firm (ij) such that $\Pi_{ij}(p_1, \ldots, p_t, d_{11}, \ldots, d_{Tm_T}, q_{11}, \ldots, q_{Tn}) =$
$\sum_{t=1}^T p_t q_{tij} - C_{ij}^{tot}(q_{1ij}, \ldots, q_{Tij})$ and where $A_{ij}(\overline{q}_{ij}) = \{(q_{1ij}, \ldots, q_{tij}) \geq 0 : \ q_{tij} \in K_{tij}, \ q_{tij} \leq D^t - \sum_{(hr)\neq(ij)} \overline{q}_{thr}, \ \Pi_{ij} \geq 0\} \cup \{\underline{0}\}$. As usual, each firm is maximizing profit within the time horizon, given other agents actions.

Definition 3.11 *An equilibrium in this market economy is a vector* $(p_1^*, \ldots, p_T^*, d_{11}^*, \ldots, d_{Tm}^*, q_{11}^*, \ldots, q_{Tn}^*)$[5] *such that*

1. $(p_1^*, \ldots, p_T^*) \geq 0$

2. $\sum_i d_{ti}^* = \sum_j q_{tj}^*$ *for all* $t = \{1, \ldots T\}$

[5] To avoid a too cumbersome notation I have re-indexed variables here

3. $(d^*_{1i}, \ldots, d^*_{Ti}) \in X_i(\overline{d^*_i})$ *and* $u_i(d^*_{1i}, \ldots, d^*_{Ti}) = max_{\underline{d} \in X_i(\overline{d}^*_i)} u_i(d_{1i}, \ldots d_{Ti})$ *for all i*

4. $(q^*_{1j}, \ldots, q^*_{Tj}) \in A_j(\overline{q}^*_j)$ *and* $\Pi(p^*, q^*_j) = max_{q_j \in A_j(\overline{q}^*_j)} \Pi(p^*, q_j)$ *for all j*

We have the following result:

Lemma 3.12 *Let us assume that each supplier production set* K_{ij} *is a compact set*[6] *and that* $C_{ij}(\underline{q})$ *is of class* $C^1(\dot{K_{ij}})$[7]. *Let* S^* *be an equilibrium. If* $q^*_{tij} > l_{tij}$ *(where* l_{tij} *is the minimum load required by firm* (ij) *to produce at time t) then* $p^*_t \geq \frac{\partial C_{ij}}{\partial q_{tij}}(q^*_{1ij}, \ldots, q^*_{Tij})$; *if* $q^*_{tij} = l_{tij} > 0$ *then* $p^*_t \geq \frac{C^V(q^*_{1ij},\ldots,l_{tij},\ldots,q^*_{Tij}) - C^V(q^*_{1ij},\ldots,0,\ldots,q^*_{Tij})}{l_{tij}}$.

Proof. The result follows since, otherwise, supplier (ij) would find more convenient, given other agents actions, to reduce production of item q_{tij} (if $q^*_{tij} > l_{tij}$) or not producing at all (if $q^*_{tij} = l_{tij}$). ■

Hence, under the hypothesis of Lemma (3.12), equilibria will be the same if the set of available actions of each supplier is further restricted by requiring market price to be higher than partial marginal costs:

$$A_{ij}(\overline{q}_{ij}) = \{\underline{0}\} \cup \{(q_{1ij}, \ldots, q_{tij}) \geq 0 : \ q_{tij} \in K_{tij}, \ q_{tij} \leq D^t - \sum_{(hr) \neq (ij)} \overline{q}_{thr}, \ \Pi_{ij} \geq 0$$

$$\begin{cases} \overline{p}_t \geq \frac{\partial C_{ij}}{\partial q_{tij}}(q_{1ij}, .., q_{tij}, ..q_{Tij}) & \text{if } q_{tij} > l_{tij} \\ \overline{p}_t \geq \frac{C^V(q_{1ij},..,l_{tij},..q_{Tij}) - C^V(q_{1ij},..,0,..,q_{Tij})}{l_{tij}} & \text{if } q_{tij} = l_{tij} > 0 \end{cases}$$

Let us consider the following optimization problem P'':

$$\max \sum_{t=1}^{T} \int_0^{Q_t} D_t^{-1}(x)dx - \sum_{i=1}^{k} \sum_{j=1}^{n_k} C^V_{ij}(q_{1ij}, \ldots, q_{Tij}) - C^F_{ij} Z_{ij} - \epsilon \sum_{t=1}^{T} \sum_{i=1}^{k} \sum_{j=1}^{n_k} z_{tij}$$

$$s.t.$$

[6]This is always the case if K_{tij} are closed intervals of the real line

[7]$\dot{K_{ij}}$ is the interior of K_{ij}

$$
\begin{aligned}
Q_t &= \sum_{i=1}^{k}\sum_{j=1}^{n_i} q_{tij} && \forall\, t = \{1,\ldots,T\} && (15)\\
p_t &= D_t^{-1}(Q_t) && \forall\, t = \{1,\ldots,T\} && (16)\\
q_{tij} &\in K_{ti} && \forall\, t,i,j && (17)\\
q_{tij} &\leq M z_{tij} && \forall\, t,i,j && (18)
\end{aligned}
$$

$$
\begin{aligned}
(p^t - \frac{\partial C_{ij}}{\partial q_{tij}}(q_{1ij},\ldots,q_{Tij}))(q_{tij} - l_{tij})z_{tij} &\geq 0 && \forall\, t,i,j && (19)\\
(p_t l_{tij} - C^V(..,l_{tij},.,) + C^V(,..0,.,))z_{tij} &\geq (q_{tij} - l_{tij}) Neg z_{tij} && \forall\, t,i,j && (20)\\
\sum_{t=1}^{T}(p_t q_{tij} - C^V_{tij}(q_{tij}) - C^F_{ij} Z_{ij} &\geq 0 && \forall\, i,j && (21)
\end{aligned}
$$

$$
\begin{aligned}
\sum_{t=1}^{T} z_{tij} &\leq M Z_{ij} && \forall\, i,j && (22)\\
z_{tij} &\in \{0,1\} && \forall\, t,i,j && (23)\\
Z_{ij} &\in \{0,1\} && \forall\, i,j && (24)\\
p_t &\geq 0 && \forall\, t = \{1,\ldots,T\} && (25)
\end{aligned}
$$

This is a mixed-integer, non-linear program where the binary variable z_{tij} is used to model activity of firm (ij) at time t, $z_{tij} = 1$, or non-activity, $z_{tij} = 0$. If firm (ij) is active at time t, i.e., $q_{tij} > 0$, then constraint (18), where M is a large positive number, will force z_{tij} to be 1. If, though, $q_{tij} = 0$ then the term $-\epsilon z_{tij}$ in the objective function will force z_{tij} to be zero in an optimal solution since ϵ is a small, positive number chosen small enough not to modify the optimal solution. The binary variable Z_{ij} is used to model activity, $Z_{ij} = 1$, or no activity, $Z_{ij} = 0$, of firm (ij) during the entire time horizon. If firm is active in any period, $z_{tij} = 1$ for some t, then constraint (22) will force $Z_{ij} = 1$ and the firm fixed cost for the entire period will be reflected in the welfare function. If the firm (ij) is not active in any period then the term $-C^F_{ij} Z_{ij}$ will force the variable Z_{ij} to be zero in an optimal solution since costs are positive numbers. As already mentioned, the objective function is the usual welfare function, i.e., maximizing the total sum of consumers surplus and firms profit. Constraints (15) and (16) equate supply and demand while (17) is each firm technological constraint on production. Constraints (19) and (20) must be satisfied by all equilibria points by lemma (3.12) while constraint (21) imposes the profitability requirement within the given time horizon. Note that constraint (19) is an identity if $z_{ij} = 0$ or $q_{tij} = l_{tij}$ while forces price to be greater or equal to partial marginal cost when $q_{tij} > l_{tij}$. Similarly, constraint (20) is an identity if $z_{tij} = 0$ and is redundant if $q_{tij} > l_{tij}$ since Neg is a large, negative number. When $q_{tij} = l_{tij}$, instead, the constraint reveals the trade off between producing at minimum load or not producing at all. Finally, when $Z_{ij} = 0$ constraint (22) is again the identity $0 = 0$ since $Z_{ij} = 0$ implies $z_{tij} = 0$ for all t which further implies $q_{tij} = 0$ for all t. As in the previous section, we could make the distinction between the case of sunk or non-sunk fixed cost to claim Pareto optimality. However, even if fixed costs are already sunk, it may still be preferable for the firm not to produce at all in time horizon T if producing, while recovering part of fixed costs, requires additional investments to cover operative deficits. From this point of view, to say that a profitability requirement in a given time horizon, for active firms, is a departure from economic efficiency is a little stretchy. Moreover, a profitability assumption is made also in Arrow-Debreu, [5], to prove the existence of equilibria in a general economy. However, for purists, we keep the distinction and claim just second best Pareto efficiency when costs are sunk. We have the following result:

Theorem 3.13 *Let us assume that each supplier production set K_{ij} is a compact set and that $C_{ij}(\underline{q})$ is of class $C^1(\dot{K}_{ij})$. Suppose that a solution S^* to problem P'' exists. If $\frac{\partial C_{ij}}{\partial q_{tij}}(q^*_{1ij}, \ldots, q_{tij}, \ldots, q^*_{Tij})$ is quasi-monotone for all t and all (ij) then S^* it is an equilibrium and it is second-best Pareto efficient among all possible equilibria.*

Proof. We have that $p^*_t \geq \frac{\partial C_{ij}}{\partial q_{tij}}(q_{1ij}, \ldots, q_{tij}, \ldots, q_{Tij})$ for all $q_{ij} \in A_{ij}(\overline{q}^*_{ij})$. Hence, $p^* d \geq \nabla C_{ij}(q_{ij}) d$ for all $d \geq 0$ and all $q_{ij} \in A_{ij}(\overline{q}^*_{ij})$ which implies that the optimal action for supplier (ij) is to fulfill all the residual demand in any market. ■

Corollary 3.14 *Let P''' be the optimization problem obtained from P'' by removing the profitability requirement (constraint 21) and let S^* be a solution to problem P'''. Under the hypothesis of Theorem (3.13) S^* is an equilibrium and it is Pareto efficient among all possible equilibria.*

Theorem 3.15 *Let us suppose that market demand in each period is a continuous function, that each supplier production set K_{ij} is a compact set, that $\lim_{q \to 1^+} D_t^{-1}(q) = +\infty$ for all t and that $C_{ij}(\underline{q})$ is of class $C^1(\dot{K}_{ij})$. If we* *can partition the time horizon T in r intervals, $T = \bigcup_{t=1}^{r} T_i$ (r can also be 1) and in each of these periods it exist a single firm such that it can profitably (i.e., the firm can recover its time horizon fixed cost in period T_s at the market prices) supply $(q_{1s}, q_{2s}, \ldots, q_{ns})$ such that $p_{ts} = D_{ts}^{-1}(q_{ts})$ is greater than partial marginal costs then a solution to problem P'' will exists.*

Proof. The hypothesis guarantee that the feasible region is non empty. For example, the existence in each period T_s of a firm that can profitably supply the market at prices above marginal cost is guaranteed if there exists a technology j_s with no minimum load requirement. In this case in fact we can solve the following problems:

$$\text{Max } C_j^T(q_1, \ldots, q_t)$$

$$s.t.$$
$$q \in K_j$$

Since C_j^T is continuous and K_j is compact a solution exists. Let us call it C_j^*. Then we will solve the problems for all $t = \{1, \ldots t_s\}$

$$\text{Max } \frac{\partial C_j}{\partial q_t}(q)$$

$$s.t.$$
$$q \in K_j$$

Again, since partial derivatives are continuous and K_j is compact, a solution will exists. Let us call $(p_1^*, \ldots, p_{t_s}^*)$ these solutions and let $p_v^* = \max\{p_t^*\}$. Set $p_v^* = \max\{p_v^*, C_j^*\}$. Consider then $q_t^* = D_t(p_t^*)$. If $q_t^* \in K_{tj}$ for all t we are done since $p^* q^* \geq C_j^*$. Otherwise it is enough to rise the prices to curtail demand until a feasible solution is obtained. Analogous to the proof of Theorem (3.6) we can bound the prices in each market to obtain a bounded feasible set. For each possible combination of active or non active firms the objective function is continuous and the feasible set is either empty or non empty and compact. In this latter case the maximum will exist and since the number of possible combination is finite the result follows. ■

When variable costs are separable in the number of periods the problem is simpler and the obvious modifications are left to the reader. Note that, since we have indexed each production set by period t as well as each quantity q_{tij} we can use exactly the same framework to find equilibrium prices in T

markets for T different products with independent demand and that can be supplied by the same firm at cost $C^T(q_1, \ldots, q_T)$, both in the single period and in the multi period setting.

4 Equilibria with Inelastic Demand

In this section we would address the case in which demand D is inelastic. This means that consumers do not have a well defined set of strategies. Would we know each consumer budget and each consumer demand, we could, at most, compute an implicit price cap as $p = \min\{\frac{B_i}{d_i}\}$. Hence, for all prices less than p each supplier strategy set will be equal to the singleton $\{d_i\}$. At higher prices some supplier strategy set will be empty and the problem will not be well defined since the demand can not be D any longer. In general, though, in real applications we do not have this information. However, assuming large enough consumers budgets, we may say that consumers surplus is maximized at the minimum price at which all demand can be satisfied. On the other hand, producers need to maximize profits so that the welfare function reduces to a min cost allocation of demand. We could, hence, use problem $P^{''}$ introduced in the previous section in which the objective function is simplified. Let us call this problem $P^{''}_{inl}$.

$$\text{Min} \sum_{t=1}^{T} p_t D_t + \sum_{i=1}^{k} \sum_{j=1}^{n_k} C^V_{ij}(q_{1ij}, \ldots, q_{Tij}) + C^F_{ij} Z_{ij} + \epsilon \sum_{t=1}^{T} \sum_{i=1}^{k} \sum_{j=1}^{n_k} z_{tij}$$

$$s.t.$$

$$D_t = \sum_{i=1}^{k} \sum_{j=1}^{n_i} q_{tij} \qquad \forall\ t = \{1, \ldots, T\} \tag{26}$$

$$q_{tij} \in K_{ti} \qquad \forall\ t, i, j \tag{27}$$

$$q_{tij} \leq M z_{tij} \qquad \forall\ t, i, j \tag{28}$$

$$(p^t - \frac{\partial C_{ij}}{\partial q_{tij}}(q_{1ij}, \ldots, q_{Tij}))(q_{tij} - l_{tij}) z_{tij} \geq 0 \qquad \forall\ t, i, j \tag{29}$$

$$(p_t l_{tij} - C^V(.., l_{tij}, .,) + C^V(, ..0, .,)) z_{tij} \geq (q_{tij} - l_{tij}) Neg z_{tij}\ \forall\ t, i, j \tag{30}$$

$$\sum_{t=1}^{T} (p_t q_{tij} - C^V_{tij}(q_{tij}) - C^F_{ij} Z_{ij} \geq 0 \qquad \forall\ i, j \tag{31}$$

$$\sum_{t=1}^{T} z_{tij} \leq MZ_{ij} \qquad \forall\, i,j \tag{32}$$

$$z_{tij} \in \{0,1\} \qquad \forall\, t,i,j \tag{33}$$

$$Z_{ij} \in \{0,1\} \qquad \forall\, i,j \tag{34}$$

$$p_t \geq 0 \qquad \forall\, t = \{1,\ldots,T\} \tag{35}$$

When we have a single period, a producer will offer a quantity q if and only if the market price is higher than both the marginal cost and the average total cost at q. The above optimization problem has a solution if total capacity is greater or equal to demand in each period and producers do not have minimum load requirement. Since we are assuming free entrance the condition is basically equivalent to a no minimum load requirement for at least one technology. In the case of minimum load constraints for all technologies a solution may fail to exist so that existence is contingent to the specific instance of the problem. This is summarized in the following:

Lemma 4.1 *If $\sum_{i=1}^{k}\sum_{j=1}^{n_i} M_{tij} \geq D_t$ for all $t = \{1,\ldots,T\}$, where $M_{tij} = max\{K_{tij}\}$, and $l_{tij} = 0$ for all t,i,j then problem $P^{''}_{inl}$ has a solution. If $\forall\, t$ $\exists\, k_t$ such that $n_{k_t}M_{tk} \geq D_t$ and $l_{k_t} = 0$ then problem $P^{''}_{inl}$ has a solution.*

Under the usual assumptions on suppliers cost functions we also have that each supplier is maximizing profit over the set of its available strategies, given other players actions.

Lemma 4.2 *Under the assumption of Lemma (3.13), if a solution to problem $P^{''}_{inl}$ exists, then it is an equilibrium and it is second best Pareto efficient among all possible equilibria.*

Let us call $P^{'''}_{inl}$ the problem obtained from $P^{''}_{inl}$ by removing the profitability constraint.

Corollary 4.3 *Under the assumption of Lemma (3.13), if a solution to problem $P^{'''}_{inl}$ exists then it is an equilibrium and it is Pareto efficient among all possible equilibria.*

5 Efficient Energy-Only Markets

In this section we will first apply our findings to numerical examples taken from the literature, and then we will sketch an efficient energy-only market design able to restore incentives for the investors.

We will start with a very simple example from O' Neill et al., [28]. They consider a single commodity market in which all firms have the same cost structure and there are no barrier to entry. Basically, they are considering a single technology and a large number of equal firms within it. Each firm must incur a fixed cost of 1 to produce any positive amount in the range $(0, 1]$ while marginal cost is zero. The market demand curve is known and it is $p = 2 - 0.6Q$. They claim that in this economy there can not be any market equilibrium. Their reasoning is as follow: *" For any price less than* 1*, no firm will produce and there will be a shortage. For any price strictly greater than* 1*, quantity supplied is infinite and there is a surplus. Finally, for* $p = 1$*, quantity demanded is* 1.67 *but the quantity supplied will be no more than* 1*, because if a second firm enters, it will not earn enough revenue to cover its fixed cost".* Here, the trick to solve the problem is to realize that in a competitive market there is always an optimal number of firms to satisfy efficiently a given demand. In fact, the firm criterion to enter a given market is not just "positive extra-profits" but "positive extra-profits high enough" to cover its own investment costs. Note also that this is a necessary but not sufficient condition. In this example it happens that the optimal number of firm is just one, i.e., the level of demand and the technology cost structure characterize a natural monopoly. If we apply our modified Tatonnement process we get that for prices less than one no firm will supply, at price 1 all firms supply set will be the singleton 1, i.e., firms will produce an amount equal to 1 or will not produce at all since otherwise they would not cover their investment cost. But the quantity demanded by the market at price 1 is only 1.6666 so that at this price there can not be any match between demand and supply. Hence, the auctioneer increases the price, let say to 1.1. The market demand is now 1.5 but each firm will supply only in the interval $[0.91, 1]$ so that again there can be no match between supply and demand. Similarly, if the price is, let say 1.32, the market demand is 1.1333 and each firm will supply anything within the interval $[0.7575, 1]$ and again no match is possible. When the price is raised each firm supply set will enlarge while market demand will decrease. Finally when the price is 1.4 the market demand is 1 and each firm supply set is $[0.7142, 1]$. Now a match is possible with one single firm supplying all the market. If we increase the price we see that the welfare function decreases so that the price 1.4 is the equilibrium price. Note also that this is the price that a monopolist will choose to maximize its own profit. The extra-profit earned by this single firm is not high enough to support a new entrant and the market is in equilibrium.
We proceed with another example from Scarf, [31], reported also in [28].

There are two technologies, Smokestack and High Tech. The first has large capacity, is moderately inexpensive to construct per unit of capacity and has fairly high marginal cost of production. The second technology has medium capacity, is expensive to set up for unit of capacity but has lower marginal cost of production. Data are summarized in table 1.

Table 1: Smokestack versus High Tech

	Smokestack	High Tech
Capacity	16	7
Construction Cost	53	30
Marginal Cost	3	2
Average Cost at Capacity	6.3125	6.2857

Suppose that we want to satisfy an inelastic demand of 61. The optimal mix of technologies and firms within each technology will be to construct 3 Smokestack plants and 2 High Tech plants, all running at capacity except for one of the Smokestack plant producing 15 units. Equivalently, we can have the same mix and the same cost with Smokestack plants all producing 15.6667. This is the usual assumption when we talk of equal firms within the same technology: they will produce the same output. If we solve problem P''_{inl} with this data we get that the equilibrium price is (about) 6.3830. At this price Smokestack plants will earn zero profits while the more efficient technology will earn the difference between the market price and its own average total cost at capacity (times the capacity, i.e., the amount produced). We will call these profits "infra rents", following Joskow, [24]. In this case they are quite low, i.e., 0.6808 for each high tech plant or 1.3617 for the High Tech technology as a whole. Hence, with the proposed approach, we do find the expected economics dynamic, found in any microeconomics textbook: marginal technology earning zero profit and more efficient technologies earning infra rents.

Our last example is more targeted towards the energy industry and is taken from Joskow, [24]. There are three hypothetical electric generating technologies with different capital cost to operating cost ratios and a hypothetical load duration curve representing the number of hours during the year the aggregate system demand or "load" reaches any particular demand level. Demand is at least $10,000$Mw for the entire year (8760 hours) and reaches a peak of $22,000$Mw in just one instant. Demand between these two values is reached according to the following load duration curve

$$D = 22,000 - 1.37H$$

where H represents the number of hours system load reaches a level D. In table 2 we summarize the cost data.

Table 2: Production Costs of Generating Technologies

Generation Technology	Annualized Capital Costs ($/Mw/Year)	Operating Costs ($/Mw/Hour)
Base Load	240,000$	20$
Intermediate	160,000$	35$
Peaking	80,000$	80$

The least cost mix of generating technologies, the number of hours in which each technology is marginal, total cost and total revenues by pricing at marginal costs are displayed in table 3

Table 3: Least Cost Mix of Technologies

Generation Technology	Capacity Hours	Marginal	Tot. Cost	Tot. Revenues
Base Load	14,694	5333-8760	5,940,379,300 $	4,765,153,180$
Intermediate	4,871	1778-5333	1,385,604,660$	995,973,370$
Peaking	2,435	1-1778	367,977,200$	173,177,200$
Total	22,000		7,693,961,160$	5,934,303,750$

Hence, by pricing at marginal cost none of the technologies is able to recover total cost and the shortfall is exactly $80,000$$Mw has predicted by theory, [7]. If we apply our pricing model to this optimal mix we find that price is equal to marginal cost for the entire year except for one hour a year, i.e., at peak load, when it needs to raise to $80,085.6356$$ in order for all technologies to break even. Note that the price we find, namely $80,085.6356$$, is slightly higher than the Ramsey-Boiteux price of $80,080$$ since we are applying this price for an hour in which demand is not always at $22,000$ but between $21,999$ and $22,000$. We will have anyway that the marginal technology will be making zero profit while the more efficient technologies will be earning infra rents. They are anyway quite low, specifically $376,690$$ for the base load, corresponding to 0.006% of its total cost, and $76,161$$ for the intermediate, corresponding to 0.005% of its total cost. If we had more peak load during the year the peak load price would decrease. For example, with the same mix of technologies, if we had 2 hours during the year in which demand were between $21,999$ and $22,000$ the peak load price would be $40,075$$ and the marginal technology would be the intermediate, earning 0 profits, while the base load would earn $146,940\$$, a $0,0025\%$ infra rent, and the peaking would earn $59,335\$$, a $0,016\%$ infra rent. Hence, knowing the number of time the system is at peak is important. We could also spread the peak load period, to get lower prices that are at trade off, though, with total cost and hence are "inefficient". Still, there can be other type of "valuation": political, social, psychological, etc., which would call for "inefficient" but lower prices. We could make an analogy with debt installments, in which a buyer is willing to pay overall more in order to have a lower installment value. In table 4 we report different length for the peak load period, the corresponding prices and the infra rents (and extra-profits) for the efficient technologies.

Table 4: Peak Load Period Length versus Price

Demand cut off	Peak Period (hours)	Peak Price (Mwh)	Base Load Profits	Intermediate Profits	Peaker Profits
21,999	1	80,085.6356 $	376,690 $	76,161 $	0 $
21,972.05	20.4	4,024.21326 $	7,081,781 $	2,298,871 $	0 $
21,178.06	600	240.419537 $	239,096,686 $	79,210,849 $	0 $
21,041,07	700	222. 320645 $	288,655,570 $	95,639,413 $	0 $
20,934.22	778	211.650134 $	329,789,260 $	109,275,061 $	0 $

These results would sustain the view of those who are contrary to price caps or of those who sustain electrical systems should be a regulated monopoly. For example, a regulated monopoly could still price in such a way that each generating unit is able to recover its own costs from its own revenues (hence, getting the right price signals and avoid stranded costs), spreading the peak to get lower prices for customers and to use extra-profits for base research on sustainable energies or to give it entirely back to customers. This is probably just a matter of political taste and it is of no concern to us.

Next, we will present a very simple, energy-only market design that price at peak load and, hence, it is able to recover fixed costs.

- Every year the market maker will forecast yearly demand, (or for some period T into the future) and will ask generators, existing and incumbent, to bid for this demand both their (expected) variable cost and the fixed cost that they need to recover within the year (period).
- The market maker will solve problem P'' or P''_{inl} (depending on the demand elasticity) to get the expected peak load price and above all to determine the fixed cost that each technology needs to recover within

the period. The technology fixed cost is computed by gathering together similar firms who have been selected in the optimization program to supply and summing their bid fixed cost. Any ties will be broken by selecting existing versus incumbent generation, i.e., given the same bid, priority will be given to existing generation in the selection process. This is to give potential new entrants the right signals.

- Once this yearly auction is over and the market maker has the proper information the day ahead energy market will proceed as usual, with the market maker receiving bid and ask and matching supply and demand to clear the market. The market maker will solve problem $P^{''}$ (or $P^{''}_{inl}$) where, though, the profitability constraint, (21), has been removed along with the fixed cost in the objective function. Hence, we just have producers bidding, for each price, supply sets made of all quantities whose marginal cost is below or equal to the cried price. Basically, this turn to be pricing at marginal cost. While the day-ahead market clears every day, the market maker will compute infra-rents which accrue to the efficient technologies and will adjust consequently the fixed cost that each technology has to recover. Hence, fixed cost for each technology is a function of time, $F_i(t)$ and can only decrease.

- When the system is at peak load at any time t the market maker will compute the peak load price based on $F_i(t)$ and on the expected number of peaks during the remaining period.

At the end of the year (period) the proposed system guarantees that technologies have recovered, through infra-rents or peak load prices, the fixed costs determined in the auction. It is important to realize that this sort of auction is not a forward market since selected suppliers are not given any contract nor are they sure to be selected to supply during the year. This process is needed to get cost information from generation and to allow incumbents to have the right signals as to what type of technology and capacity is really needed by the system. Hence, the auction is not taking away the normal business risk! Existing generation has not incentives to lower fixed cost bids as a deterrent for incumbents since then the peak load price set by the system operator would not cover their true costs. They might, though, bid higher. In a true competitive system this behavior should be offset by incumbent bids. However, in order to prevent possible gaming, the market maker could set caps based on its knowledge of the cost of each technology capacity (again, here the assumption of perfect information!). Gathering this type of cost information is usually easier than gathering information about $VOLL$ (Value of Lost Load) as many proposed scheme in the literature require. Also, since the proposed day ahead market is basically based on the classical theory of marginal cost pricing, there is no need to modify the way existing day-ahead market work. Hence, applying this scheme to existing market is not very costly since, furthermore, the yearly auction does not require any contract signing or exchange of money but just an exchange of information. It is just a way to gather private information and make it public. We would discuss at length pros and cons of such a system, taking into consideration system constraints, in our forthcoming paper.

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